and we deduce

$$\frac{m_a - h_a}{a} \geq \frac{(b-c)^2}{2a^2}$$

so we find

$$\sqrt{\frac{m_a - h_a}{a}} \geq \frac{\sqrt{2}}{2}\left|\frac{b-c}{a}\right|$$

If we write similar inequalities and adding them, we deduce the inequality of the statement.

Nicuşor Minculete

Second solution. Lemma.

$$am_a \geq 2\triangle + \frac{(b-c)^2}{2}. \tag{1}$$

Proof. Firstly note

$$a^2 m_a^2 = \frac{2a^2b^2 + 2c^2a^2 - a^4}{4} =$$

$$= \frac{2a^2b^2 + 2b^2c^2 + 2c^2a^2 - a^4 - b^4 - c^4 + b^4 + c^4 - 2b^2c^2}{4} =$$

$$= \frac{16\triangle^2 + \left(b^2 - c^2\right)^2}{4} = 4\triangle^2 + \frac{\left(b^2 - c^2\right)^2}{4}.$$

Then

$$am_a \geq 2\triangle + \frac{(b-c)^2}{2} \Longleftrightarrow a^2 m_a^2 \geq 4\triangle^2 + 2\triangle(b-c)^2 + \frac{(b-c)^4}{4} \Longleftrightarrow$$

$$\Longleftrightarrow 4\triangle^2 + \frac{\left(b^2 - c^2\right)^2}{4} \geq 2\triangle \cdot (b-c)^2 + \frac{(b-c)^4}{4} \Longleftrightarrow$$

$$\Longleftrightarrow \left(b^2 - c^2\right)^2 \geq 8\triangle \cdot (b-c)^2 + (b-c)^4$$

and we have

$$\left(b^2 - c^2\right)^2 - 8\triangle \cdot (b-c)^2 - (b-c)^4 = (b-c)^2\left((b+c)^2 - (b-c)^4 - 8\triangle\right) =$$

$$= 2(b-c)^2\left(\frac{bc}{2}-\triangle\right) \geq 0$$

because $\triangle \leq \frac{bc}{2}$.

Using inequality (1) we consequtively obtain:

1. $\sum_{cyc} am_a \geq \sum_{cyc}\left(2\triangle + \frac{(b-c)^2}{2}\right) = 6\triangle + a^2+b^2+c^2-ab-bc-ca;$

2. Since $\frac{m_a-h_a}{a} = \frac{am_a-ah_a}{a^2} = \frac{am_a-2\Delta}{a^2} \geq \frac{(b-c)^2}{2a^2}$ then

$$\sum_{cyc}\sqrt{\frac{m_a-h_a}{a}} \geq \sum_{cyc}\sqrt{\frac{(b-c)^2}{2a^2}} = \frac{\sqrt{2}}{2}\sum_{cyc}\left|\frac{b-c}{a}\right|.$$

Arkady Alt

W59. (Solution by the proposer.) First notice that some of the squares will be parallel to the sides of the lattice board. There will be squares of sizes 1×1, 2×2..., 2017×2017. For each $1\leq n\leq 2017$ the number of $n\times n$ squares is $(2017-n+1)^2$. To find them all, we just have to add

$$S = (2017-1+1)^2 + \cdots (2017-2017+1)^2 = 1^2+\cdots+2017^2 = 2737280785.$$

There will also be squares whose sides are not parallel to the grid.

To estimate their number, one can consult the diagrams shown in Figure 1. For every pair (m,n) such that $2\leq m+n\leq 2017$ and $m,n\geq 1$, the polygon having coordinates $(m,0),(m+n,n),(n,m+n),(0,n)$ are all squares. For each pair (m,n) with one has $(2017-(m+n)+1)^2$ squares that can be obtained by translation (note that m and n can be equal or different). The answer to the question is given by the formula

$$S = \sum_{0\leq m+n\leq 2017,\, m,n\geq 0}(2017-(m+n)+1)^2 =$$

$$= \sum_{k=1}^{2017} k(2018-k)^2 = \sum_{k=1}^{2017}(2018-k)k^2 = \sum_{k=1}^{2017}(2018k^2-k^3) = 1381984890721.$$